Aeroelastic Stability of Cylindrical Shells Subjected to a Rotating Flow

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It is experimentally demonstrated that a cylindrical shell subjected to a rotating flow may become dynamically unstable. The flutter motion takes the form of a circumferentially traveling wave. The data confirm qualitatively earlier theoretical predictions, although further refinement of the theoretical modeling of the fluid motion is desirable in order to improve the quantitative agreement between theory and experiment.

Nomenclature

a = inner shell radius
b = outer shell radius
d = tube diameter
E = modulus of elasticity of inner shell
f = frequency
h = thickness of inner shell
L = cylinder length
l_t = tube length
m = number of axial half waves
n = number of circumferential waves
r = radial coordinate
$\bar{V}_A$ = average axial velocity determined from mass flow measurement
V_{TO} = linear, circumferential fluid velocity at outer shell
V_T = linear, circumferential fluid velocity at radial position r
v = wave velocity
w = inner shell transverse displacement
T = time
x = axial coordinate
λ = wavelength
θ = circumferential coordinate
θ_T = tube alignment angle

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Index categories: Aeroelasticity and Hydroelasticity; Airbreathing Propulsion, Subsonic and Supersonic.

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I. Introduction

Physical Nature of the Problem

NUMEROUS investigations have been carried out in the past to determine the aeroelastic stability characteristics of a cylindrical shell element whose outer surface is exposed to fluid flow along its longitudinal axis. However, examples do occur in industrial application where the flow conditions on a flexible cylindrical shell element are not entirely axial; i.e., the fluid velocity may have both axial and tangential components. In jet engines of dual spool configuration, for example, swirl flow between rotating parts is not uncommon. Reference 1 describes theoretical studies to determine flutter characteristics of cylindrical shells in a rotating, swirling (vortex) flow environment. In this paper, we describe an experimental program undertaken to study this phenomenon including preliminary correlation with the theory of Ref. 1.

Schematic drawings of the major experimental apparatus are provided in Figs. 1 and 2. In Fig. 1 an experimental apparatus (the "standard model") which can be used to develop a swirl type of flow around a flexible cylindrical shell is shown. Figure 2 shows a "similar" apparatus which is a geometrically quarter-scale model of the standard apparatus. All tests are run at ambient conditions at low Mach numbers. The basic configuration consists of two concentric cylindrical shells. The outer cylinder is essentially rigid and rotates at high speed. At approximately two-thirds length, air is drawn in radially by suction through several small holes around the circumference, thereby creating a rotating vortex-type flow about the inner cylinder. The inner cylinder whose vibrations are of concern is relatively flexible but does not rotate. The goal has been to construct a model of this system and make measurements of the flow conditions and inner cylinder vibration levels. Of principal concern are the rotation speeds of the outer cylinder and the radial mass flow rates through the holes on the outer cylinder at which severe vibrations of the inner cylinder may occur.

Related Earlier Work

The present problem is broadly related to a number of similar problems which have been studied in the past which are usually grouped under the name, "panel flutter," or more formally, the aeroelastic stability of plates and shells. (See Ref. 2 for a recent survey of the literature on this subject.) The basic physical phenomenon is the dynamic aeroelastic instability or "flutter" of

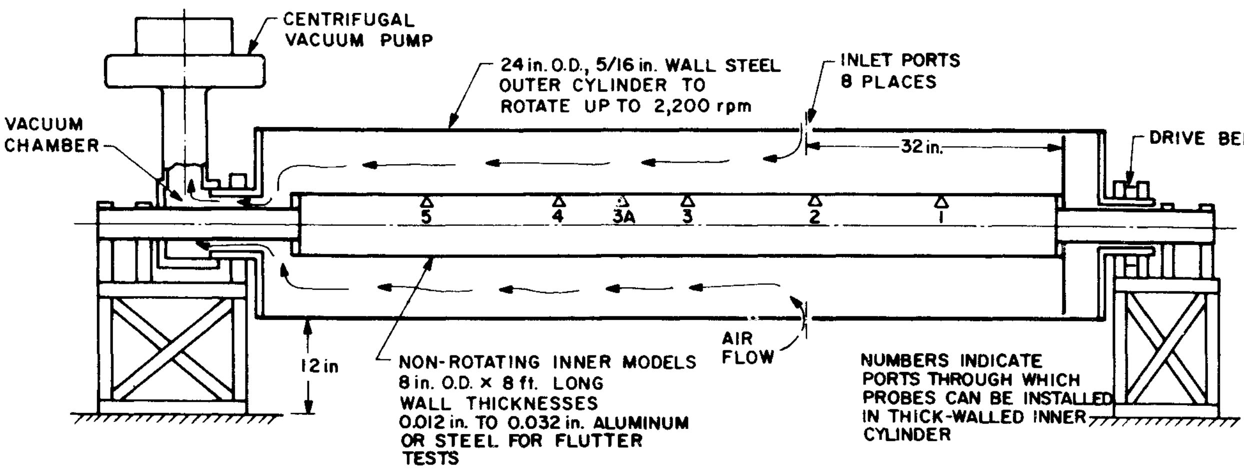

Fig. 1 Standard experimental model.

a thin plate or shell structure due to the mutual interaction of the structural motion and the surrounding fluid flow. "Flutter" occurs when the mean flow velocity exceeds a critical value, the "flutter velocity." The unusual feature of the present problem is the nature of the flowfield. The structure is conventional (a cylindrical shell), and the flutter of such structures has been studied previously when the basic flowfield is one of uniform velocity along the cylinder axis. However in the present problem the flow is of a swirling type induced by the rotation of an outer cylinder (concentric with the inner flexible flutter cylinder) and a radial inflow through holes placed around the circumference of the rotating cylinder. For some combinations of radial inflow and rpm of the cylinder, a vortex-type flowfield can be established leading to large circumferential velocities near the inner flexible cylinder which may be of sufficient magnitude to induce flutter. In principle any type of flowfield can induce flutter but a vortex-type field with velocity increasing for decreasing radius is more likely to produce the high velocities required for flutter. Few previous experimental data are available for the flowfield of interest here. Reference 3 seems to offer the closest relevant data and suggests that the combinations of rotational and radial flow required to produce a vortex-type flow can be correlated on the basis of rotational and radial Reynolds numbers. Some theoretical work on determining the basic steady-state flowfield is also contained there. (Also see the analysis in Ref. 4.) In this paper we report some preliminary experimental results for the mean velocity field.

A more complete description of the present work is contained in Refs. 5–8. Consult Ref. 8 in particular for an in-depth discussion of the flowfield. The reader may also wish to consult the interesting work of Paidoussis[9,10] on cylinders subjected to low speed axial external or internal flows.

Of course rotating machinery such as compressors and turbines are subject to several types of aeroelastic problems. The present paper treats only one particular type related to shell flutter.

II. Experiment

Experimental Apparatus

The basic designs with nominal dimensions and material are shown in Figs. 1 and 2. The major component parts for the two models are described in Refs. 5 and 8.

Inner flexible cylinder

Aluminum and steel shells, eight ft long, with 8–12 in. diam, and 0.012–0.032 in. thickness were used for the standard model tests. Polyvinyl chloride cylinders 21 in. long, 3 in. diam, and 0.004 in. thick have been tested in the quarter-scale model. The quarter-scale flutter cylinder was mounted concentric with a 2 in. stiff brass tube (see Fig. 2).

Flutter Experiments

Flutter tests have been conducted in both the standard and quarter-scale models.

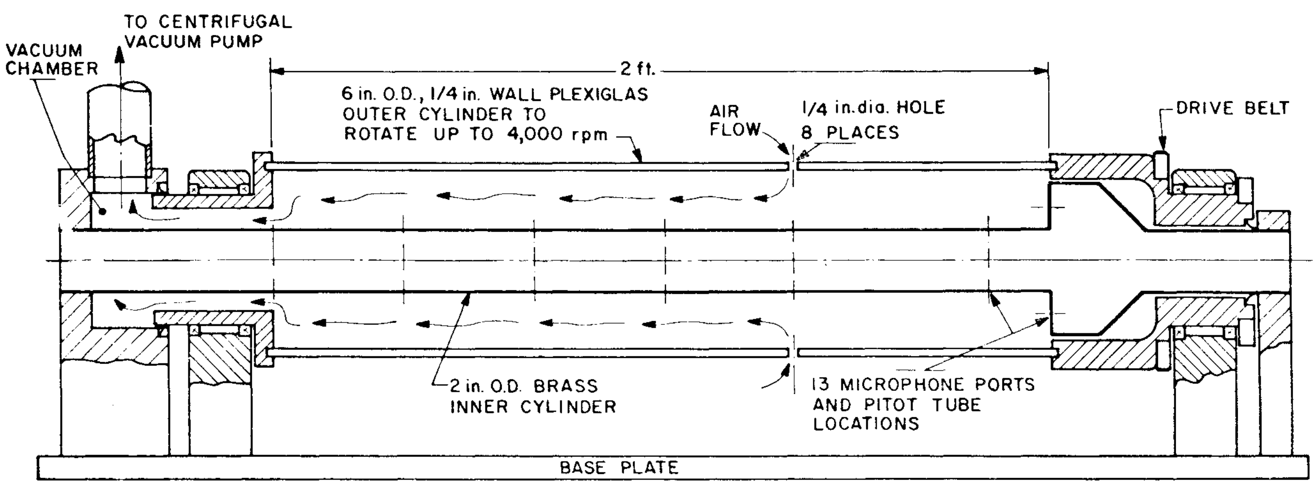

Fig. 2 Quarter scale experimental model. Polymer flutter test cylinders are mounted concentrically with brass inner cylinder.

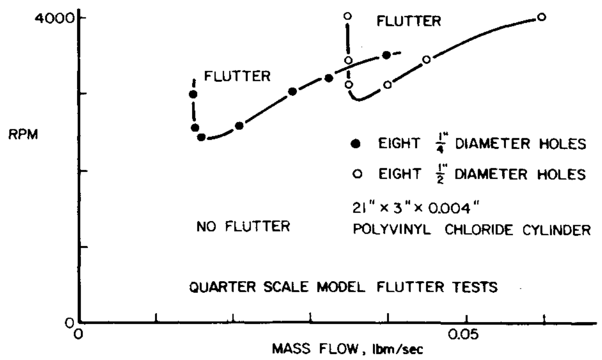

Fig. 3 Rpm vs mass flow for which flutter occurs.

Quarter-scale model

The tests are generally conducted by fixing the outer cylinder rpm at some constant value and varying the mass flow rate from zero to the maximum rate attainable. If no flutter is encountered, the rpm is increased to a higher constant value and the mass flow again varied. This process is repeated until flutter is obtained. Flutter is identified by observing the large increases in strain gauge response as displayed on an oscilloscope trace. From such tests a region may be determined in terms of rpm vs mass flow for which flutter occurs (see Fig. 3). Two results are shown for two different hole configurations of the outer cylinder. For both configurations there were eight equally spaced holes around the circumferences located 16 in. (two thirds of the length) from the exhaust end. The two configurations had holes of different diameters, $\frac{1}{4}$ in. and $\frac{1}{2}$ in., respectively. The inner cylinder was the polyvinyl chloride shell described previously.

Note that flutter occurs over a range of mass inflows above a minimum critical rpm. The smaller holes give a smaller minimum rpm of the outer cylinder. The flutter frequency was approximately 250 cps, much higher than predicted theoretically.[5] Although these tests were of a qualitative nature because of the complex annular geometry between cylinders, they were very helpful in planning, conducting, and interpreting the tests discussed below.

Standard model

The test procedure for the standard model tests was the same as that for the quarter-scale tests. However, in an early test, an inner aluminum cylinder was permanently damaged as the minimum critical rpm was exceeded. Hence, all subsequent experiments were limited to determining this rpm. This was done by fixing rpm and varying mass flow to determine the maximum rms strain. By then increasing rpm and plotting rms strain vs rpm the onset of flutter at the minimum critical rpm was readily determined.[5]

Two major series of tests were conducted. In the first a systematic variation in inlet hole geometry was studied for a

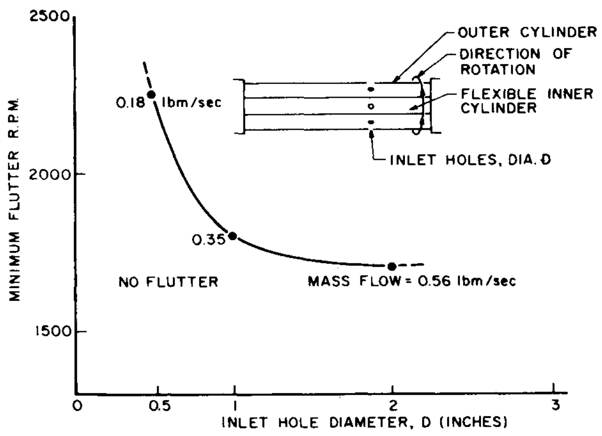

Fig. 4 Minimum flutter rpm for various hole diameters in standard model, 0.020 in. inner cylinder.

Table 1 Summary of flutter data; 1 in. diam, $\frac{5}{8}$ in. length inlet holes

Cylinder	Minimum Flutter rpm		Flutter Frequency cps		Mass Flow lbm/sec
	Theory	Experiment	Theory	Experiment	Experiment
8 in. × 0.032 in.[a] (0.032)[b]	1300	2100	44	70	0.56
8 in. × 0.020 in. No. 1[a] (0.020)	951	1500	9.5	50–100	0.36–0.56
8 in. × 0.020 in. No. 2[a] (0.020)	951	1800	9.5	65	0.25–0.44
8 in. × 0.020 in. No. 2 (0.020)	951	1300	9.5	140	0.25
8 in. × 0.016 in. (0.0165)	633	1100	9	97	0.23
8 in. × 0.012 in. (0.0135)	500	960	9	97	0.28
10 in. × 0.020 in. (0.019)	780	1400	8	86	0.21–0.36
12 in. × 0.020 in. (0.020)	776	1600	7	105	0.27
8 in. × 0.012 in. Steel (0.012)	848	800	6	68	0.25

[a] These results are for $\frac{5}{16}$ in. length hole.
[b] ( ) Indicates measured thickness.

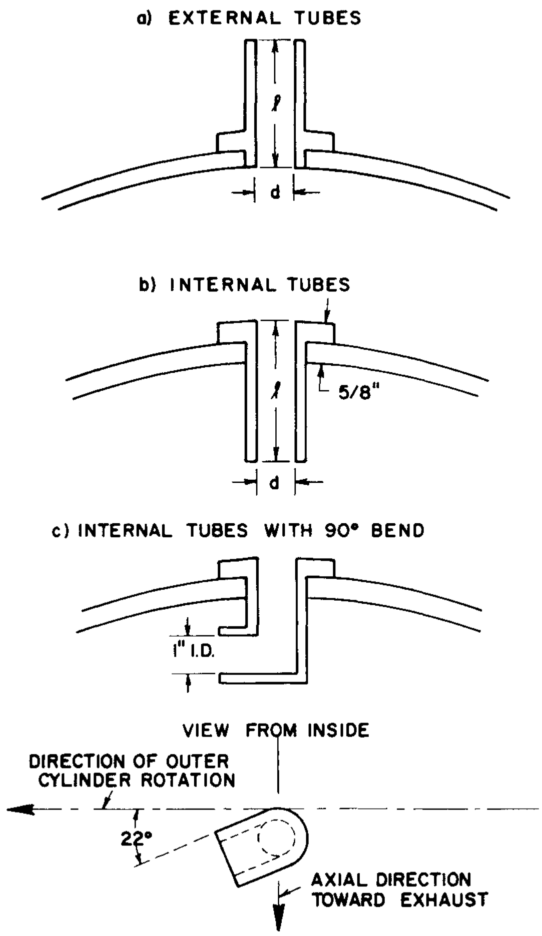

Fig. 5 Inlet configurations.

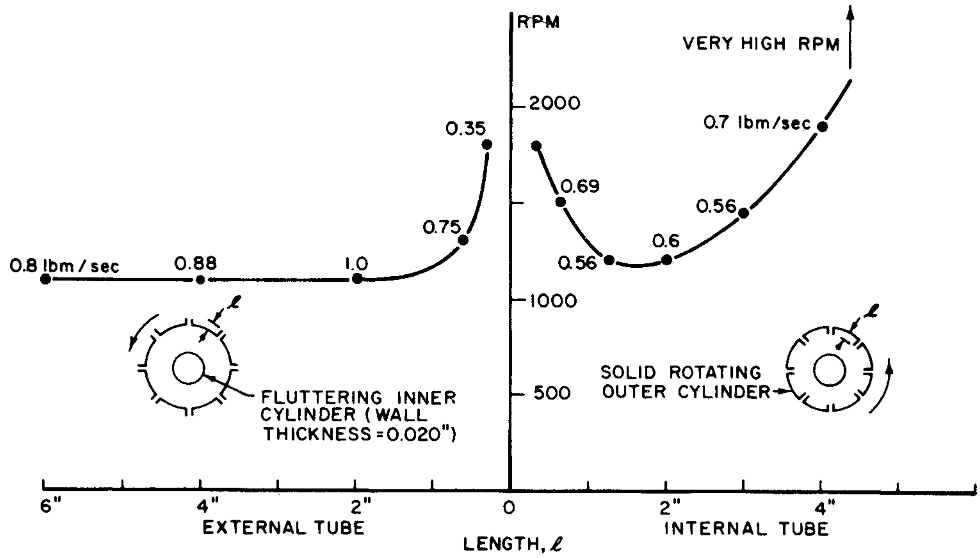

Fig. 6 Minimum flutter rpm for various inlet tube lengths (1 in. diam). l includes cylinder wall thickness of $\frac{5}{16}$ in.; hence plain port results are shown both as "external" and "internal" tubes.

single inner cylinder. In the second a systematic variation in inner cylinder parameters was undertaken for a fixed inlet hole and cavity geometry. Hence, in the first series we consider the effect of changes in the nature of the flowfield and in the second changes in inner cylinder structural characteristics. A summary of all flutter data is presented in Table 1.

Effect of Inlet Hole Geometry

In all these tests the inner cylinder was aluminum with an 8 in. diam and 0.020 in. thickness. In Fig. 4 the influence of outer cylinder hole diameter is shown. Note the expanded scale. The effect of external and internal tubes was also studied. (Tube lengths include the outer shell thickness.) The tube geometry is shown in Fig. 5 and the flutter results in Fig. 6. External tubes lower the flutter rpm but beyond a certain length there is no further reduction in flutter rpm. Internal tubes are detrimental for short lengths but beneficial for longer lengths. These flutter results are explainable in terms of the following model of the fluid mechanics.[8]

It is thought that in the case of plain port inlets, the inlet flow does not achieve the full tangential velocity of the cylinder, thus producing a vortex in the annulus characteristic of some lower tangential velocity. As the length of the ports is increased externally (external tubes), the tangential velocity of the inlet air also increases, until the tubes are sufficiently long that the inlet tangential velocity matches that of the cylinder. Any further increase in the length of the inlets should not produce a noticeable change in the flow in the annulus. Applying the same reasoning to internal tube inlets, it seems likely that short internal tubes should produce higher tangential velocities at the inlet than plain ports. However, increasing the length of internal tubes also has the effect of moving the effective location of the inlet closer to the axis of the cylinder, thus reducing the tangential velocity of the inlet itself. For long internal tubes, this effect should dominate, resulting in a net lower tangential velocities near the inner cylinder.

Lastly an internal tube with a right angle bend was used with centerlines of each section of 2 in. length. The 1 in. diam tube could be rotated through 360° and flutter rpm was measured as a function of angular orientation. The results are presented in Fig. 7. As may be seen, the most severe flutter condition is when the tube exit is aligned in the direction of outer cylinder rotation. Conversely with the tube exit in the opposite direction no flutter was observed within the rpm range available. These results were expected since it is clear that with the tube exit in the direction of outer cylinder rotation a stronger vortex-like flow would be formed and conversely for the tube directed in the opposite orientation.

Flutter Frequencies and Mode Shapes

Of considerable interest is the structural mode shape during the flutter oscillation. The 0.020 in. aluminum shell was instru-

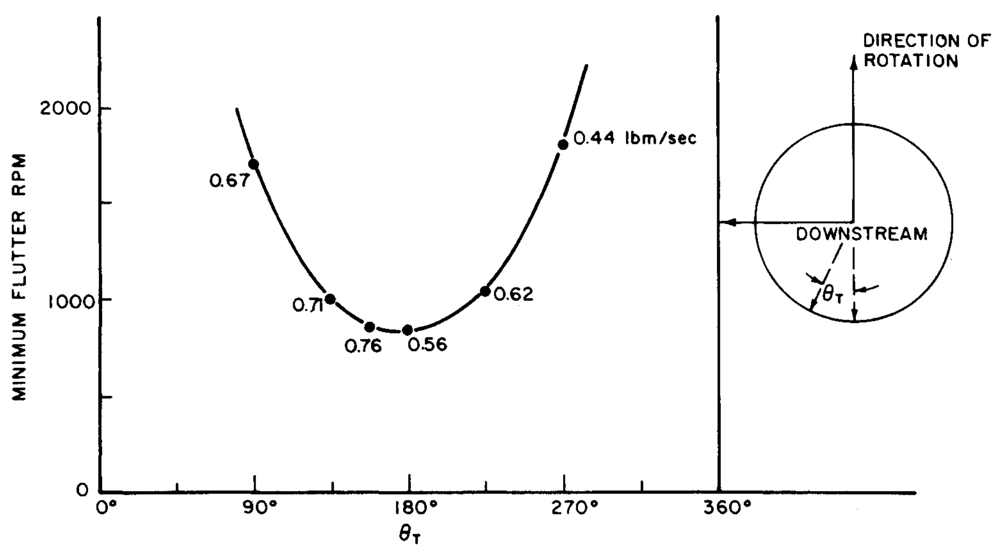

Fig. 7 Minimum flutter rpm vs orientation of 90° bend tubes.

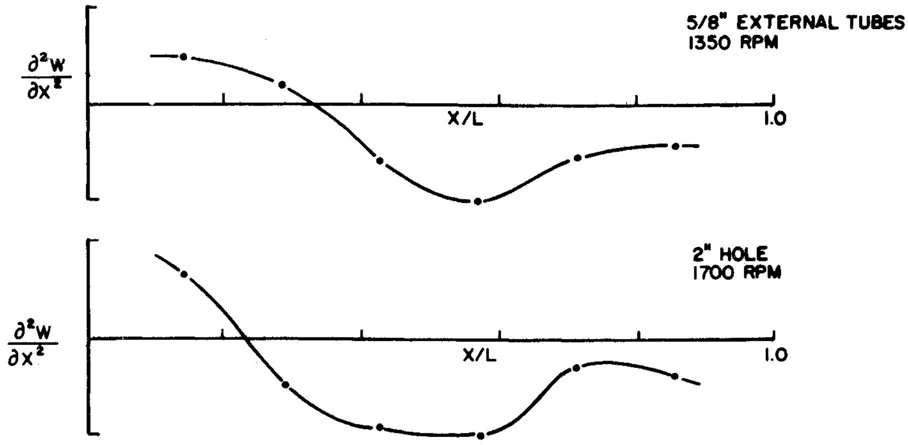

Fig. 8 Axial mode shape (curvature) during flutter.

mented with six evenly spaced strain gages along the shell axis and also six gages around a half circumference. Prior to the flutter measurements, the natural modes of the shell were determined both for their intrinsic interest as well as a check on instrumentation and experimental technique. The strain gages measure panel curvature rather than displacement in the axial direction and a combination of curvature and displacement in the circumferential direction. For a sinusoidal shape there is no difference; hence, it is anticipated no substantial difference exists between curvature and displacement as far as their circumferential variation. However, for clamped ends there will be a distinct difference axially.

For reference natural mode shapes were determined. The agreement between theory and experiment was reasonably good assuming clamped ends axially.[8]

Turning now to flutter results, measurements have been made for two different outer cylinder hole geometries, 1) $\frac{5}{8}$ in. length and 1 in. diam tubes, and 2) 2 in. diam, bare hole ($\frac{5}{16}$ in. thickness of outer cylinder). For the former axial mode shapes are shown in Fig. 8; they seem to be predominantly $m = 1$ with possibly some $m = 2$. The axial mode shapes were standing waves; the nodal points were fixed and all phase angles were either 0° or 180°. The circumferential mode shapes are shown in Fig. 9 for various times during a period of oscillation in order to demonstrate the circumferentially traveling wave character of the flutter mode. It is clear the peaks and node points move with time; for clarity we have drawn a line through the peaks for various times. We have also shown on the figure the frequency f, wavespeed v, wavelength λ, and circumferential mode number n. It was found that the two circumferential modes for the two different hole geometries are distinctly different in terms of these quantities.[8]

The flutter frequencies of the 0.020 in. inner cylinder for various hole geometries of the outer cylinder are listed in Table 2. In

Table 2 Flutter frequencies for various hole geometries

Flutter frequency	Hole geometry	
65–130 cps	Bend tube	270°
65–130		180°
55–140		90°
130	External tube	6 in.
140		4 in.
135		2 in.
140		$\frac{5}{8}$ in.
70–135	Internal tube	4 in.
55–120		3 in.
135		2 in.
62.5–125		$1\frac{1}{4}$ in.
60–135		$\frac{5}{8}$ in.

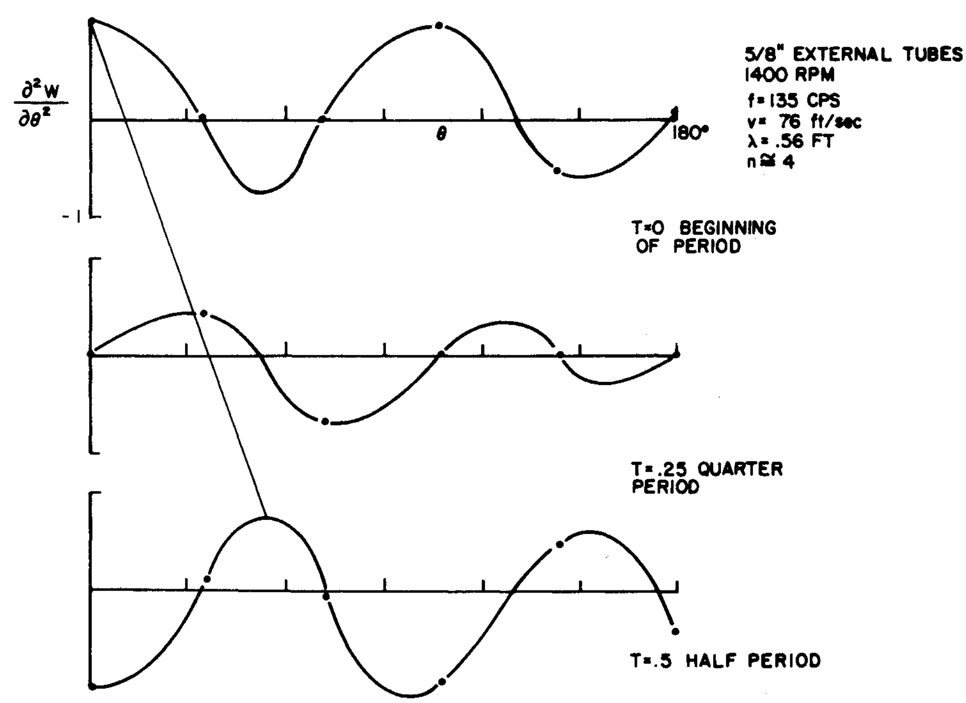

Fig. 9 Circumferential mode shape (curvature) during flutter at various times.

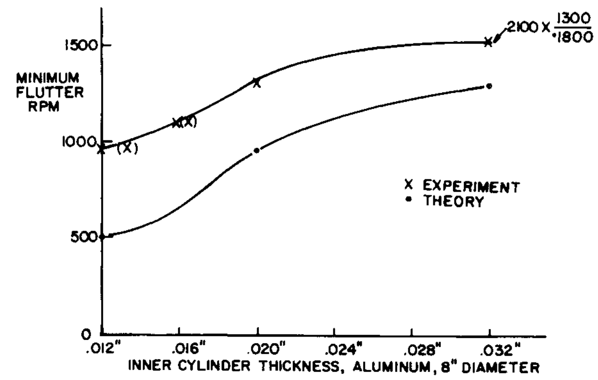

Fig. 10 Minimum flutter rpm vs inner cylinder thickness.

most cases the onset of flutter is characterized by a frequency of roughly 60–70 cps which approximately doubles to 120–140 cps as the rpm of the outer cylinder is slightly increased. However, for the external tube geometry only the higher frequency was observed. The tentative explanation for this behavior is that the lower frequency is that associated with the beginning of flutter and the higher frequency is the second harmonic of the first generated by structural nonlinearities when the inner cylinder deflections become of the order of several cylinder thicknesses. (From strain gage measurements, it is estimated the deflections have been as much as ten shell thicknesses, 0.2 in.) For the external tubes, it may be that the onset of flutter occurs in a higher circumferential mode $n = 4$, than for the other hole geometries where $n = 2$ and hence the higher frequency. It may well be that several circumferential modes have nearly the same flutter rpm and hence the flutter mode tends to be a combination of several natural circumferential modes.

Effect of Inner Cylinder Structural Parameters

Systematic variations of thickness and diameter as well as one variation in material were studied. All results are for the standard cavity geometry with inlet holes of 1 in. diam and $\frac{5}{8}$ length.

Thickness

Experimental results were obtained for aluminum cylinders of 8 in. diameter for thicknesses from 0.012 in.–0.032 in. The minimum flutter rpm is plotted vs thickness in Fig. 10. Also shown are theoretical results which will be discussed subsequently. The 0.032 in. result was obtained for a $\frac{5}{16}$ in. rather than a $\frac{5}{8}$ in. length inlet hole. It was corrected to the $\frac{5}{8}$ in. length result using the data from the 0.020 in. cylinder which were obtained for both inlet hole lengths (see Table 1).

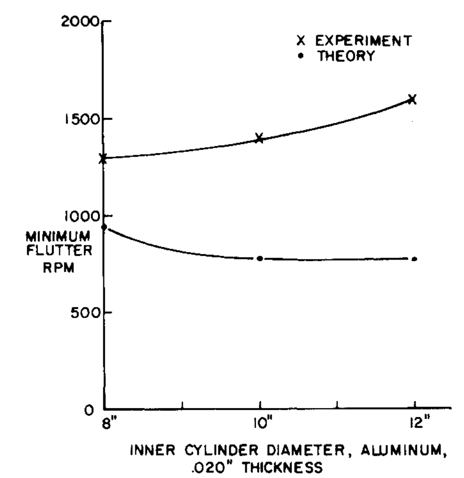

Fig. 11 Minimum flutter rpm vs inner cylinder diameter.

Diameter

Experimental results were obtained for aluminum cylinders of 0.020 in. thickness and diameters of 8 in., 10 in., and 12 in. These are shown in Fig. 11. The theoretical results will be discussed later.

Material

A steel cylinder of 8 in. diam and 0.012 in. thickness was tested. Its minimum flutter rpm and frequency along with that of all the above cylinders are shown in Table 1. Theoretical results are also given for future reference.

Correlation with Theory

There are several types of comparisons one can make between theory and experiment.

1) Compare structural natural mode frequencies of inner cylinder to assess adequacy of theoretical structural model.

2) Compare experimentally measured flutter rpm at which flutter first occurs (minimum flutter rpm) to theoretically predicted flutter rpm based upon an assumed ideal potential flow vortex mean velocity profile. The emphasis here on the minimum rpm at which flutter first occurs is a consequence of the limitation in the presently available theory to an ideal potential vortex.[1] In point of fact the mass flow through the system a) changes the nature of the mean flow, such as velocity profile shape, maximum velocity, etc., as well as b) providing an axial velocity component. The ideal theory[1] can account for b) but calculations have shown the effect is small. a) requires a substantial modification to the theory to account for arbitrary velocity profile shapes (as measured) and is apt to be more important. It is essential if one wishes to determine the complete flutter boundary in terms of rpm vs mass flow as well as account for flutter rpm variations with inlet hole geometry.

3) When a more realistic fluid mechanical model becomes available, compare theory and experiment for various inlet hole configurations and account for effect of mass flow effects.

Here we shall necessarily limit ourselves to 1) and 2) as far as quantitative comparisons, though some qualitative remarks can be made concerning 3).

1) Measured and calculated natural frequencies were compared for the lower circumferential modes for all inner cylinders used in the flutter experiments. Generally there is reasonable agreement between theory and experiment, with the theoretical values usually falling somewhat below the measured results. (See Fig. 12 for a typical result.) Basically the structural model appears adequate.

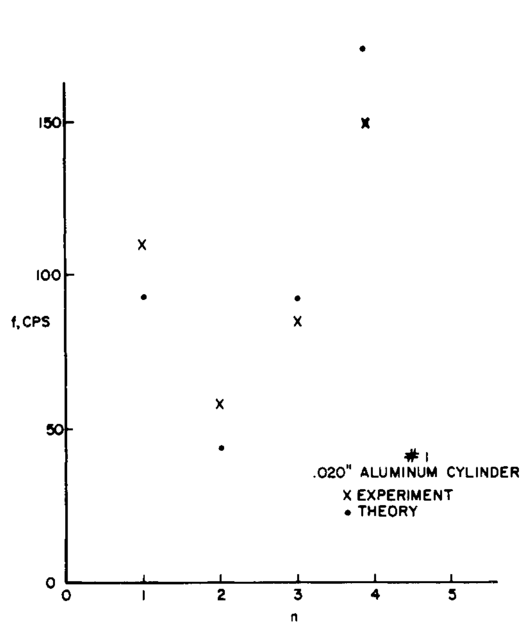

Fig. 12 Circumferential mode number.

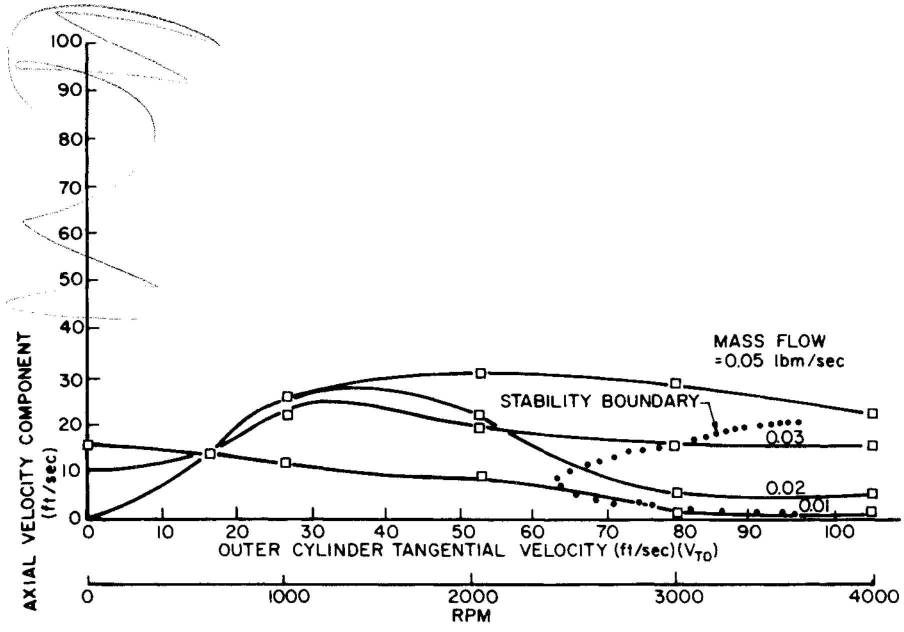

Fig. 13 Axial velocity component at radius of 1.1 in. vs outer tangential velocity.

2) As can be seen from Figs. 10 and 11 and Table 1 the theoretical minimum flutter rpm is generally smaller than the measured value. (The one exception is the steel cylinder which had a measured fundamental natural frequency approximately 20% *below* the theoretical value used in the flutter calculation.[8]) This is probably primarily a result of the idealized potential vortex flow assumed in the theory overestimating the actual mean flow velocity obtained experimentally. A second reason is the somewhat higher measured cylinder natural frequencies as compared to the theoretical values for most cylinders. In particular, for Fig. 11, the different trends for flutter rpm may be due to the experimentally observed increase in natural frequency with diameter vs the essentially constant predicted natural frequency.[8] Also it is noted that the measured and calculated flutter frequencies are in much less satisfactory agreement than flutter rpm. This has also been observed for similar problems in earlier studies.[11] Generally speaking the agreement between theory and the experiment is reasonable given the uncertainties which remain regarding the exact nature of the flowfield. In the theoretical calculations the fluid velocity at the outer cylinder is taken as 0.83 times the cylinder velocity to account for the imperfect vortex formation in a gross way.

3) Finally, modifications to the outer cylinder inlet hole configuration have led to changes in flutter rpm consistent with simple physical reasoning based upon how such modifications should enhance or diminish the strength of the vortex-type flow. These trends are supported by the flow measurements of Refs. 6 and 8.

Fluid Flow Experiments

Measurement of velocity profiles

The mathematical model of Ref. 1 which has been used to calculate the onset of flutter assumes an ideal potential vortex fluid motion. It is desirable, however, to measure the actual mean velocity profiles that include fluid rotationality.

Limited but informative results were obtained with a hot-wire technique at an axial distance 10.5 in. from the downstream end and a radial distance 0.1 in. from inner cylinder of the *quarter scale model*.[6] Axial and tangential velocity components are shown in Figs. 13 and 14, respectively. Also superimposed is the flutter boundary as determined in terms of mass flow and outer cylinder rpm (see Fig. 3). In the region where flutter occurs, the tangential velocity is 4–10 times the axial component. In subsequent measurements using a static pressure technique[6] it was assumed that the tangential velocity component dominated. Nondimensional radial profiles of tangential velocity are shown in Fig. 15. V_{TO} is the outer tangential velocity determined by the outer cylinder rotation. $\bar{V}_A$ is the average axial velocity deter-

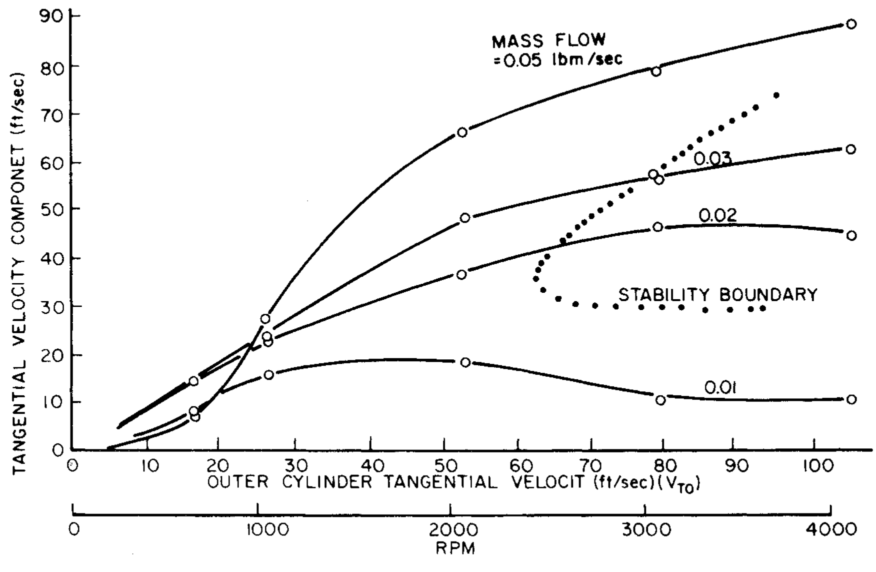

Fig. 14 Tangential velocity component at radius of 1.1 in. vs outer tangential velocity.

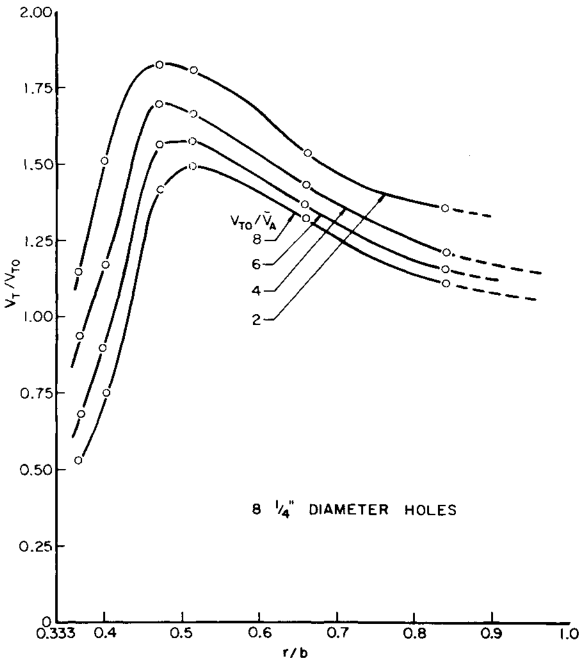

Fig. 15 Radial profiles of tangential velocity.

mined by the mass flow rate divided by fluid density times annulus cross-sectional area. As can be seen, the maximum velocity occurs near the inner cylinder with larger velocities for larger $V_{TO}/\bar{V}_A$.

McLean[8] has subsequently performed an extensive set of velocity profile experiments on the standard apparatus and developed nondimensional scaling laws for the flowfield and in a more limited sense for flutter. These results will be reported separately.

III. Conclusions and Recommendations

1) An extensive set of flutter data has been obtained and compared with theory assuming an ideal potential flow vortex velocity profile.[1] Agreement is fair with the minimum theoretical flutter rpm falling consistently below the measured values. Flutter frequency, experiment vs theory, is in poor agreement but this is anticipated for problems of this type due to the theoretically predicted rapid variation of frequency with rpm near the flutter boundary.[1,11]

2) It is strongly recommended that a systematic set of flutter calculations be conducted using the measured velocity profiles.[6,8] Such calculations, using a shear flow aerodynamic model,[12] should give improved correlation with the measured flutter data. It should be emphasized, however, that the simpler ideal potential flow vortex theory is useful in that it is normally conservative, i.e., predicts a lower flutter rpm than that measured (see Table 1). Hence, it is presently being used for design calculations.

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